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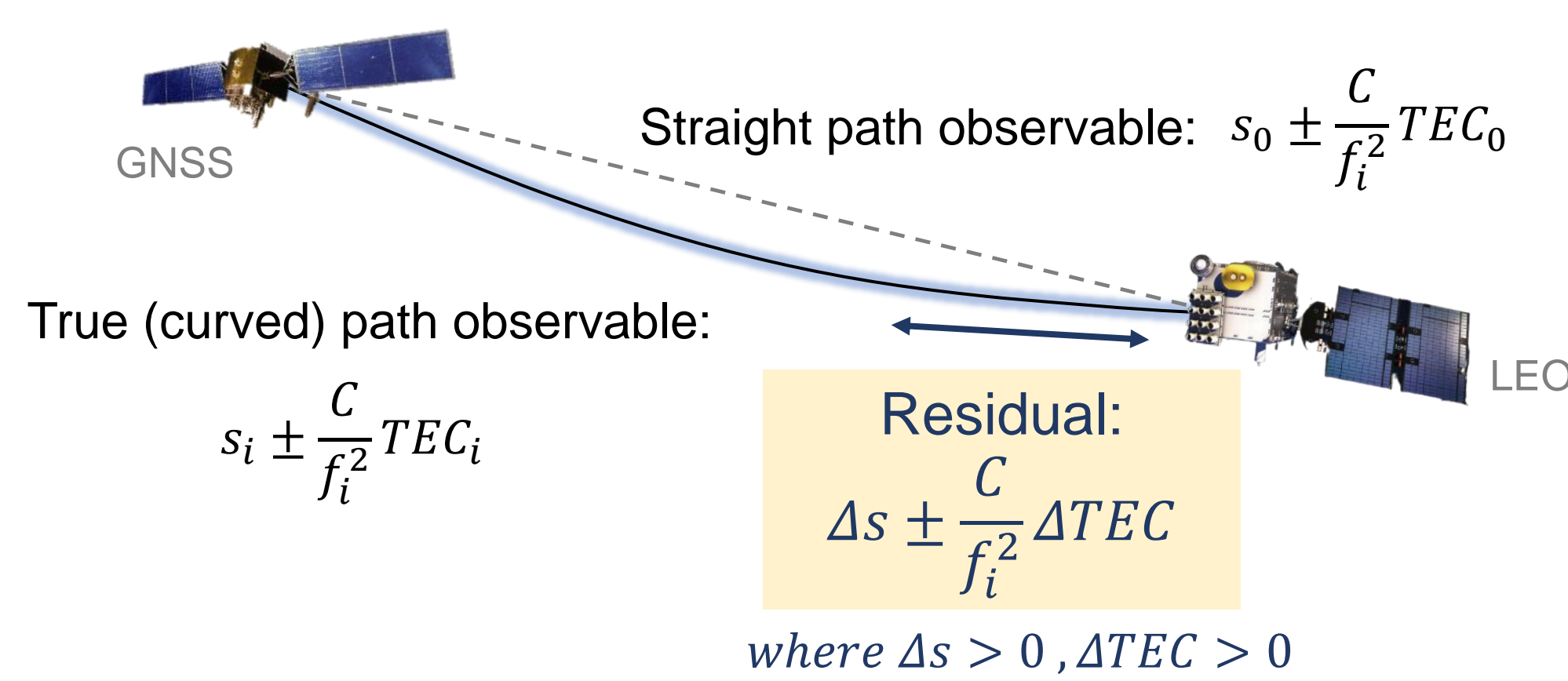
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Abstract

- Dual-frequency Total Electron Content (TEC) estimation assumes straight-line propagation, resulting in a residual bending error due to ionospheric refraction/dispersion effects.
- In this study, residual bending errors in absolute TEC were assessed using Constellation Observing System for Meteorology, Ionosphere, and Climate-2 (COSMIC-2) RO data, accounting for ionospheric bending effects on both phase and pseudorange observables.
- Residual bending error was estimated directly from measurements without a priori information on the electron density.
- Analysis across Solar Cycle 25 showed a dependency of the residual bending error on solar activity and local time, and confirmed that higher vertical TEC gradients result in larger errors, with extreme cases exceeding 20-30 TECU.

Theory

- The residual due to ionospheric ray path bending in phase/pseudorange observables consists of two terms: excess path length (Δs) and excess TEC (ΔTEC).



- Notations:
 - Ray path distance: $s = \int ds$
 - TEC: $TEC = \int N_e ds$
 - Subscript i for signal frequency
 - (+) sign for group delay, (-) sign for phase advance
- * Higher-order terms in the refractive index are neglected

\Rightarrow Residual bending term in pseudorange is larger than in phase measurements

- Using series expansions, residual bending terms in phase (L_i) and pseudorange (P_i) can be approximated as below (for details see Syndergaard (2000)):

$$L_i = s_0 - \frac{c}{f_i^2} TEC_0 - \frac{1}{2} \frac{c^2}{f_i^4} \Gamma$$

$$P_i = s_0 + \frac{c}{f_i^2} TEC_0 + \frac{3}{2} \frac{c^2}{f_i^4} \Gamma$$

\Rightarrow Under the spherical symmetry assumption, $\Gamma = D \left(\frac{dTEC_0}{da} \right)^2 + a \frac{d}{da} \int_0^a N_e^2 ds$

- Real phase measurement \hat{L}_i contains an integer ambiguity which induces a bias \hat{B}_L in the phase (relative) TEC. This can be removed by phase-to-pseudorange leveling:

$$\overline{TEC}_{12} = \frac{f_1^2 f_2^2}{C(f_1^2 - f_2^2)} (\hat{L}_1 - \hat{L}_2 - \hat{B}_L) \quad \text{where } \hat{B}_L = \langle \hat{L}_1 - \hat{L}_2 + \hat{P}_1 - \hat{P}_2 \rangle_w$$

* $\langle \rangle_w$ is the average over an arc with weighting factor w

- Then, the residual bending error in absolute TEC consists of (Høeg et al., 1998):

- Residual bending error in relative TEC, from phase dispersion
- Leveling error, from phase and pseudorange dispersion

$$\overline{TEC}_{12} = TEC_0 + \underbrace{\frac{c}{2} \frac{f_1^2 + f_2^2}{f_1^2 f_2^2} \Gamma}_{\text{①}} + \underbrace{\left(\frac{c(f_1^2 + f_2^2)}{f_1^2 f_2^2} \Gamma \right)}_{\text{②}}$$

* Multipath effects, differential code bias, and random errors are not considered

- Utilizing the frequency dependence (f^{-4}) of bending terms in phase path observables, we can estimate Γ and subsequently the residual bending error terms by taking a linear combination of dual-frequency excess phase measurements (Syndergaard, 2002).

$$\Gamma = \frac{2f_1^2 f_2^2}{c^2} \left[\frac{f_1^2 (L_1 - s_0) - f_2^2 (L_2 - s_0)}{f_1^2 - f_2^2} \right]$$

Data Processing

- The estimation of residual bending error, using the approach outlined above, requires ionospheric excess phase processing as it applies to neutral atmospheric sensing (Schreiner et al., 2009).

Real phase measurement:

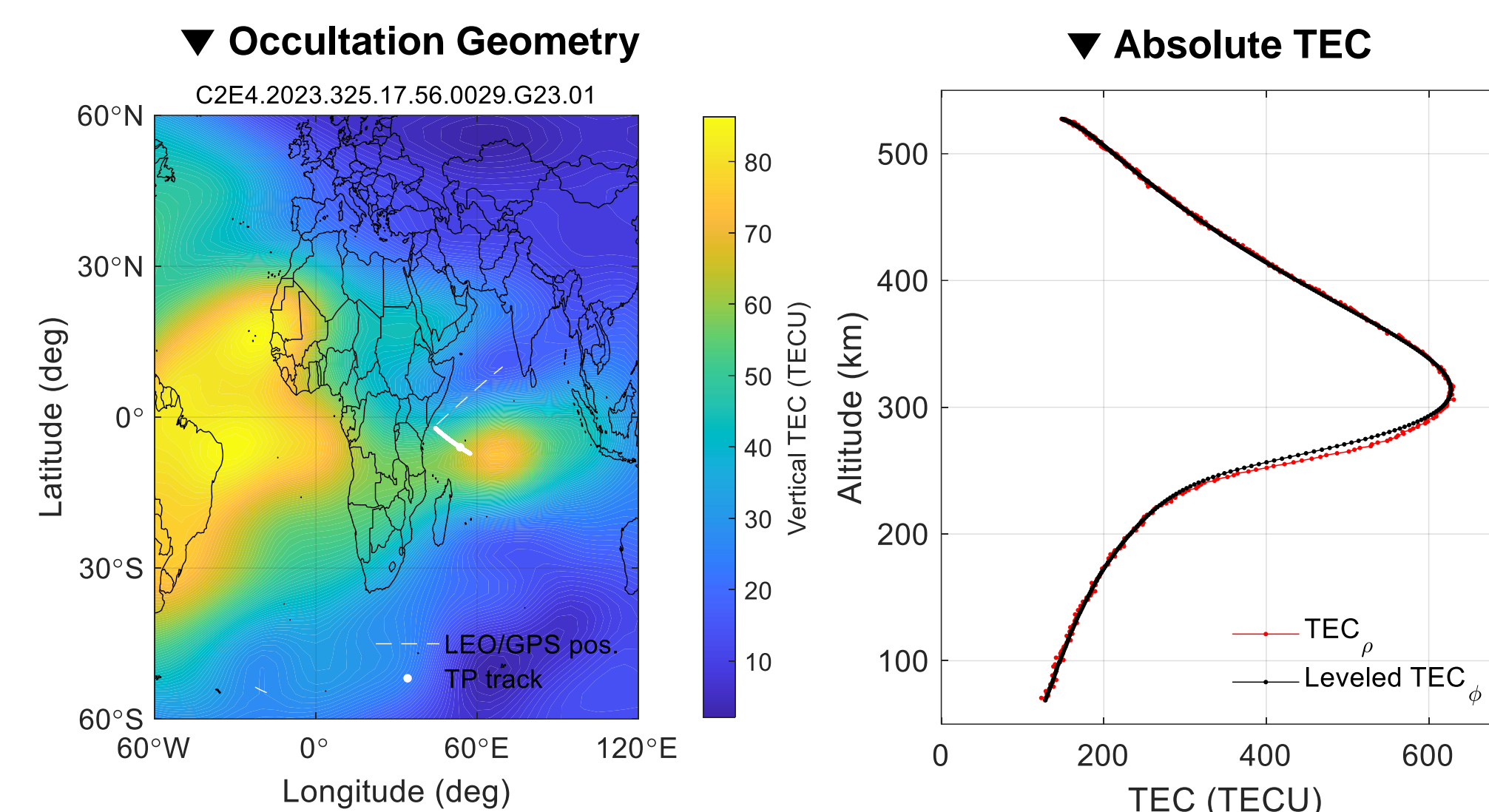
$$\hat{L}_{r,i}^s = \underbrace{\rho_r^s}_{\text{Geometric range}} + \underbrace{\delta \rho_{r,rel}^s + c \delta t_{rel}^s}_{\text{Relativistic effects}} + \underbrace{c \delta t^s}_{\text{Ionospheric excess phase}} + \underbrace{(c \delta t_r + c \delta t_{r,rel})}_{\text{Ionospheric excess phase}} - I_{r,i}^s$$

- LEO clock bias** – Single-differencing with simultaneously tracked reference and occulting GNSS satellite observations
- GNSS clock bias** – High-rate (5-sec sampling interval) GNSS clock estimates from Center for Orbit Determination in Europe (CODE)

Results

- COSMIC-2 RO measurements collected over 37 days between 2020-2024, covering periods of both low and high solar activity, were used to assess the residual bending error in dual-frequency TEC.

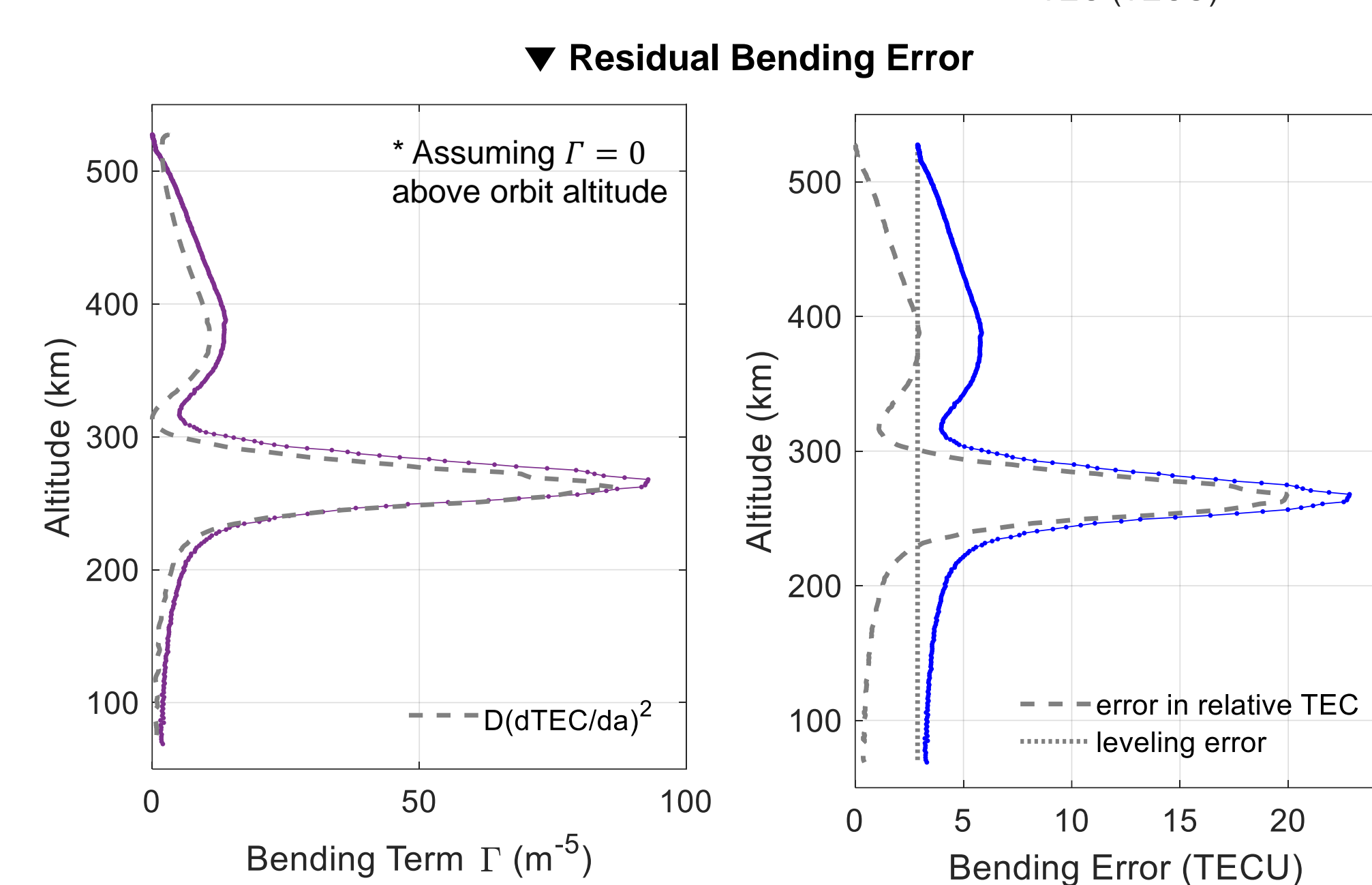
Example – High Solar Activity (2200 LT)



Input data

File type	Description
leoAtt (Level 1a)	LEO attitude information
podCrX (Level 1a)	1Hz POD observation
leoOrb (Level 1b)	LEO precise orbit determination solutions

- * GPS only
- * GPS precise orbits & clocks from CODE



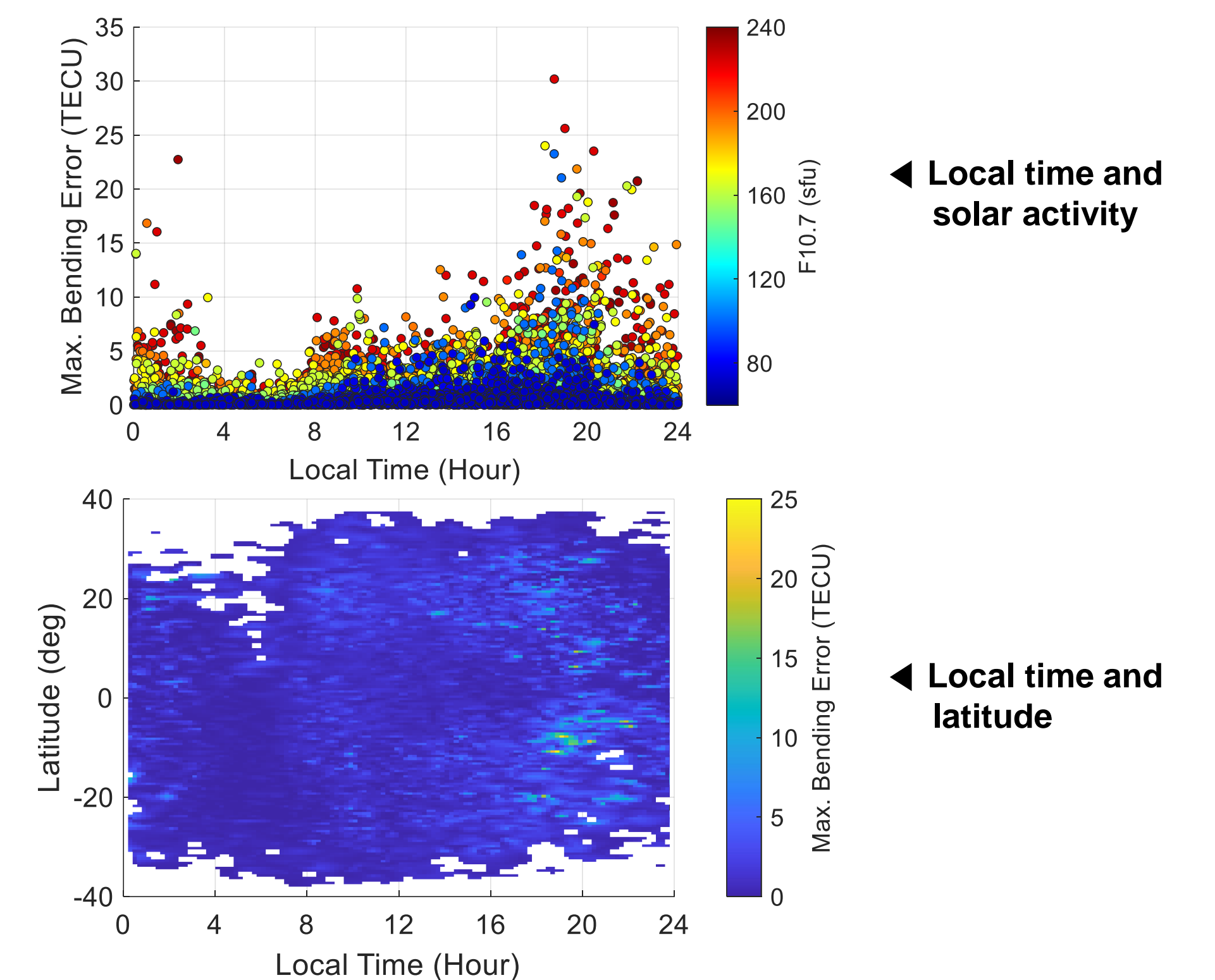
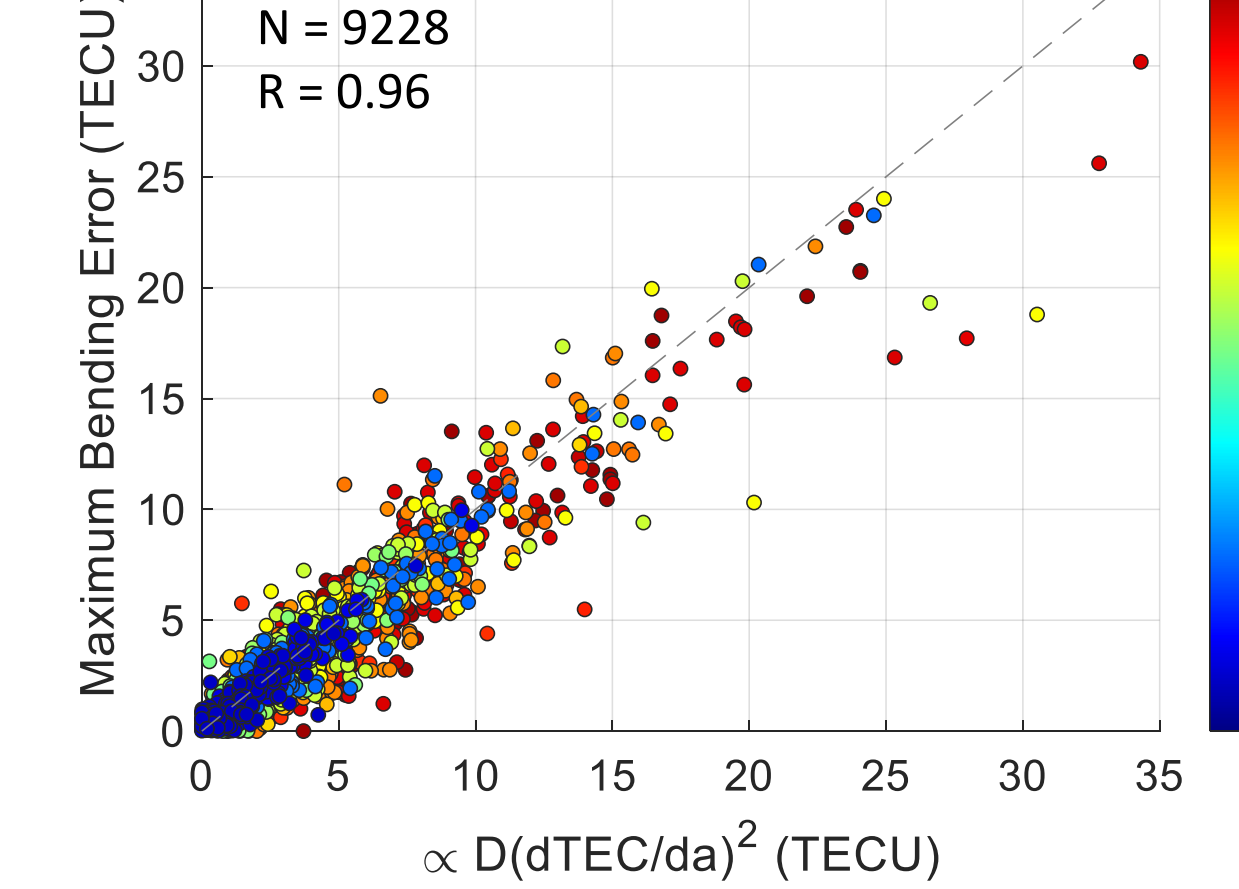
- Bending term Γ is mainly dependent on the squared vertical TEC gradient.
- Residual bending error extends up to 22.8 TECU, at maximum vertical TEC gradient (6.61 TECU/km) just below the F region peak.
- Leveling error due to dispersion is 2.88 TECU, but may vary depending on the arc length and weighting factor.
- Bending error in relative TEC generally dominates the total bending error.

Statistical Study

* Bending error in relative TEC only

Vertical TEC gradient and solar activity

* Each dot represents the maximum bending error for each occultation event



- Large bending errors occur more frequently during periods of increased solar activity, as indicated by a higher F10.7 solar flux index.
- Large errors are concentrated around 1700-2200 LT, which may be related to the sharp spatial gradient of electron density at the evening terminator due to pre-reversal enhancement (PRE).

Conclusion

- This work marks a new attempt in assessing the residual bending error in absolute TEC using real RO measurements, in contrast to prior research efforts that relied primarily on simulation studies.
- Residual bending errors were shown to be substantial under high solar activity conditions, with the largest error in our dataset exceeding 30 TECU.
- The results indicated a significant dependency of the residual bending error on vertical TEC gradient and local time.
- Under conditions inducing high TEC gradients, residual bending error correction must be implemented in order to satisfy the accuracy requirement (3 TECU) for absolute TEC.

Acknowledgments

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