Improving stratospheric gravity wave tomography from GPS-RO

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Thanks to Travis O’Brien, M. Joan Alexander, Bill Randel

NASA/Scott Kelly

Traditional wavenumber k, l determination

- Three nearby vertical profiles
  - Schmidt "strict" (100 km–250 km, 15 minutes)
  - Alexander "fair" (200 km–300 km, 20 minutes)
  - "Quarter-period" (200 km–300 km, ~4 hours)
- Wavelet decomposition
- Treat profile (and fluxes) as a sum over vertical wavenumber m
- Phase ϕ and amplitude A at every z and m

Schmidt, T., P. Alexander, and A. de la Torre (2016), Stratospheric gravity wave momentum flux from radio occultations, *J. Geophys. Res. Atmos.*, 121, 4443–4467, doi:10.1002/2015JD024135.

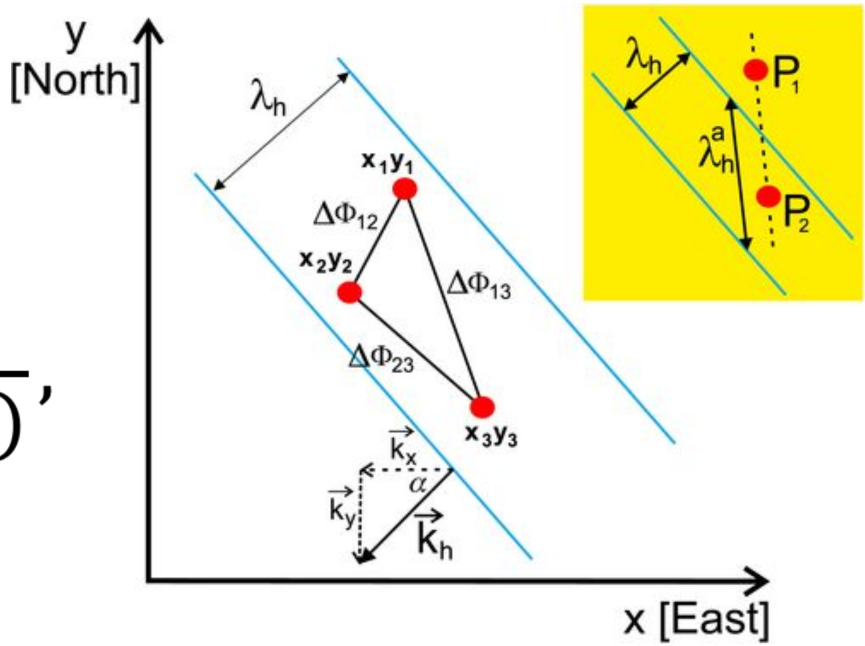
Alexander, M. J. (2015), Global and seasonal variations in three-dimensional gravity wave momentum flux from satellite limb-sounding temperatures, *Geophys. Res. Lett.*, 42, 6860–6867, doi:10.1002/2015GL065234.

Traditional solutions

- $$\Delta\phi_{21} = k(x_2 - x_1) + l(y_2 - y_1)$$
  $$\Delta\phi_{31} = k(x_3 - x_1) + l(y_3 - y_1)$$

$$\Rightarrow k = \frac{\Delta\phi_{31}(y_2 - y_1) + \Delta\phi_{21}(y_3 - y_1)}{(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)},$$

$$l = \frac{\Delta\phi_{31}(x_2 - x_1) + \Delta\phi_{21}(x_3 - x_1)}{(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)}$$

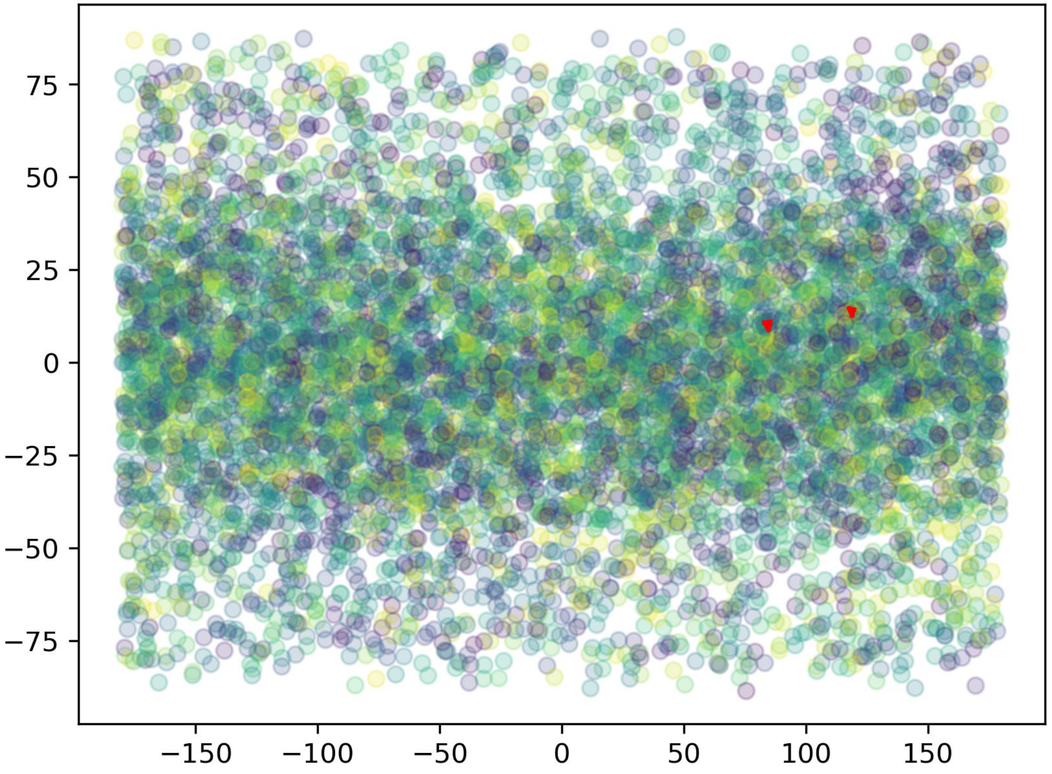


Schmidt et al., 2016

Traditional limitations

– Triads limited to tens of minutes apart
– Finite decomposition can step over "physical" wavenumber*
– Wave direction ambiguous
– Susceptible to aliasing
– Difficult to evaluate goodness of fit

Alexander (fair) cutoff

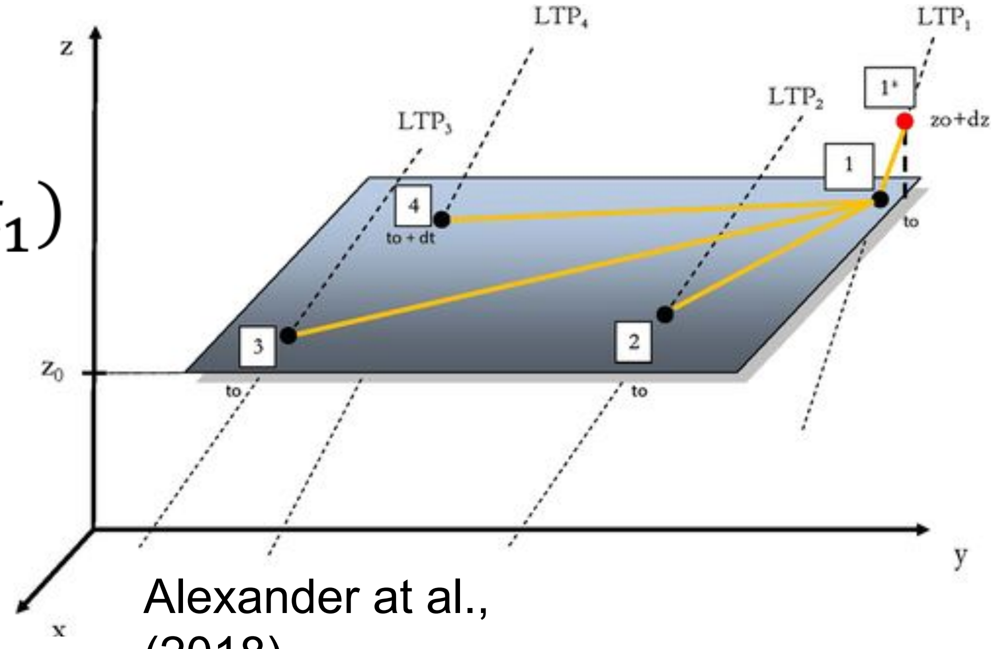


2 COSMIC2 triplets on December 31, 2020

P. Alexander et al. (2018) add a 4th profile…

$$\Delta\phi_{21} = k(x_2 - x_1) + l(y_2 - y_1)$$
$$\Delta\phi_{31} = k(x_3 - x_1) + l(y_3 - y_1)$$
$$\Delta\phi_{41} = k(x_4 - x_1) + l(y_4 - y_1) - \omega(t_4 - t_1)$$

Calculate m using two points in the same profile



Alexander at al., (2018)

Alexander P., Schmidt T., & de la Torre, A. (2018). A method to determine gravity wave net momentum flux, propagation direction, and “real” wavelengths: A GPS radio occultations soundings case study. *Earth and Space Science*, 5, 222–230. https://doi.org/10.1002/2017EA00034

Peter Alexander et al. (2018), error estimation

- $$\hat{\omega}^2 = \frac{N^2 \lambda_z^2}{\lambda_h^2}$$
- $\hat{\omega}$ included doppler shift from mean winds
- “We used then an assessment of minimum rms to evaluate the accomplishment of a dimensionless version of dispersion relation (7) along the whole altitude range and so keep the most appropriate of the four branches.”

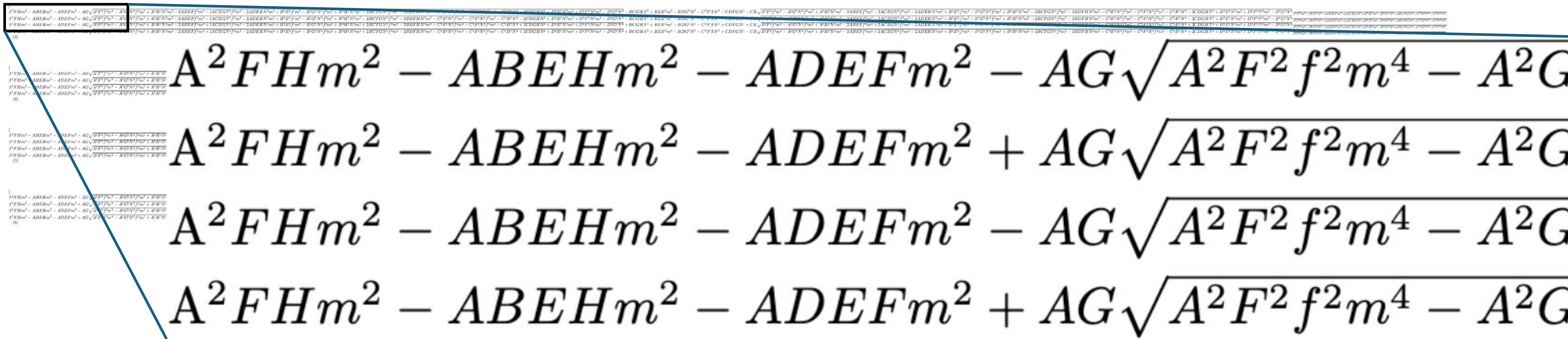
P. Alexander (2018) – limitations

- *Triads* limited to tens of minutes apart
- Finite decomposition can step over “physical” wavenumber
- ~~wave direction ambiguous~~
- aliasing
- difficult to evaluate goodness of fit
- a fourth profile required
- one retrieval per triad

Another algebraic approach

- $$\overset{D}{\Delta\phi_{21}} = k\overset{A}{(x_2 - x_1)} + l\overset{B}{(y_2 - y_1)} - \sqrt{f^2 + N^2\frac{k^2+l^2}{m^2}}\,\overset{C}{(t_2 - t_1)}$$

$$\overset{H}{\Delta\phi_{31}} = k\overset{E}{(x_3 - x_1)} + l\overset{F}{(y_3 - y_1)} - \sqrt{f^2 + N^2\frac{k^2+l^2}{m^2}}\,\overset{G}{(t_3 - t_1)}$$

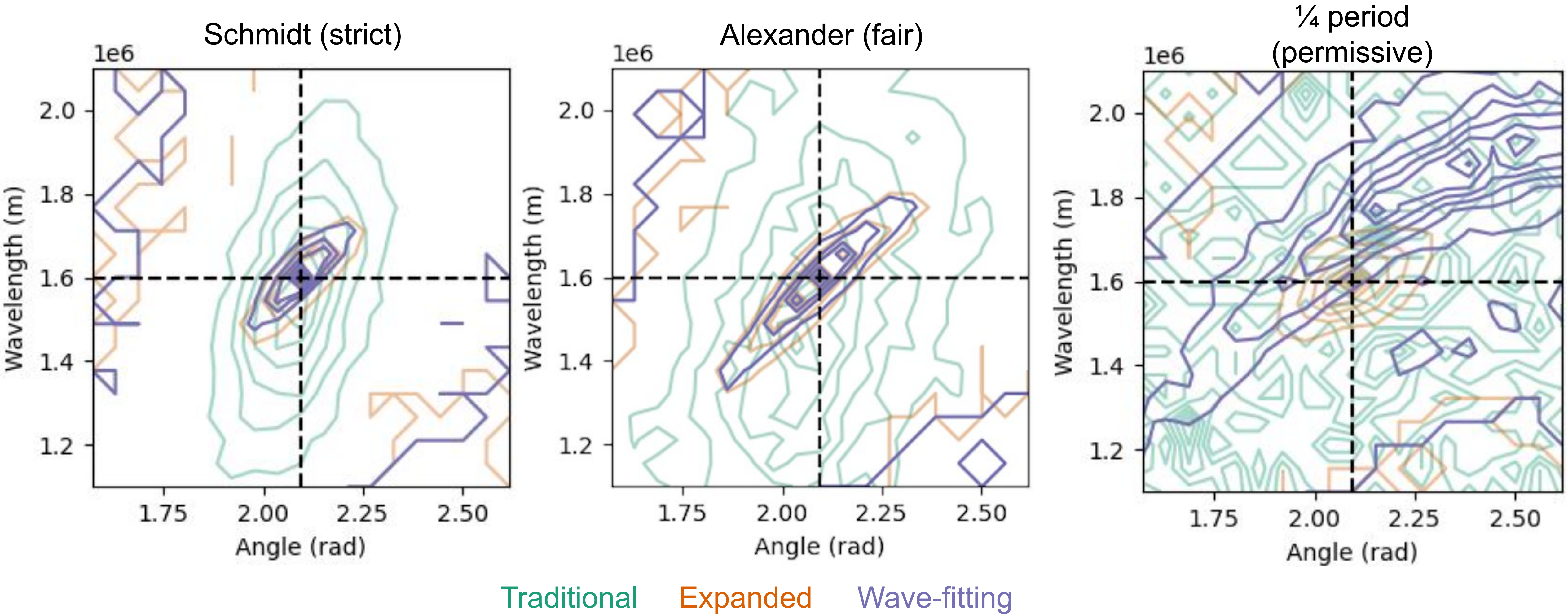


But how to choose?

- Pick the k, l pair that best fits the temperature profiles

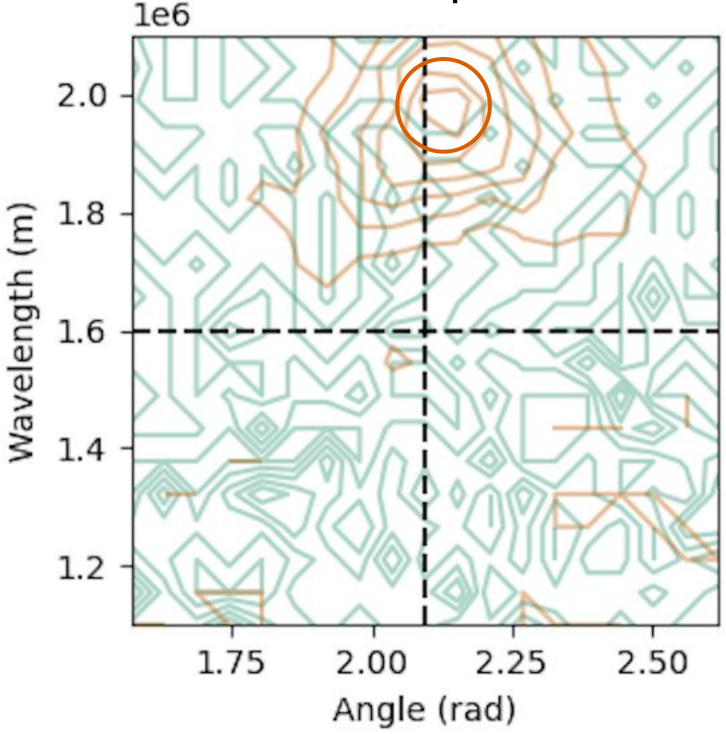
$$\mathrm{T}_{\mathrm{m,wave}} = A\cos\left(kx + ly + mz + \sqrt{f^2 + N^2\frac{k^2 + l^2}{m^2}}\,t\right),$$

Fitting a perfect plane wave, ϕ_0 known



unknown ϕ_0 makes error evaluation harder

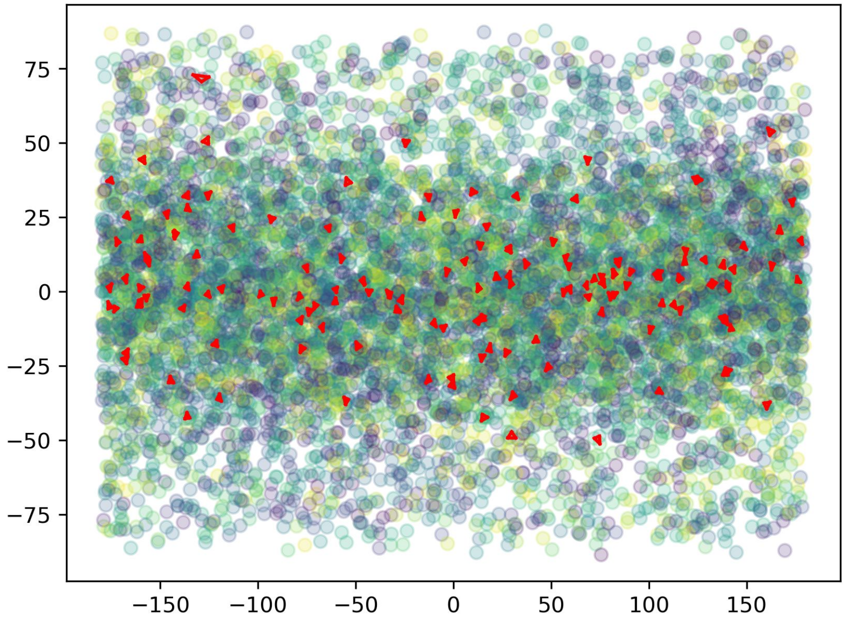
Quarter-period



λ_h biased 25% long

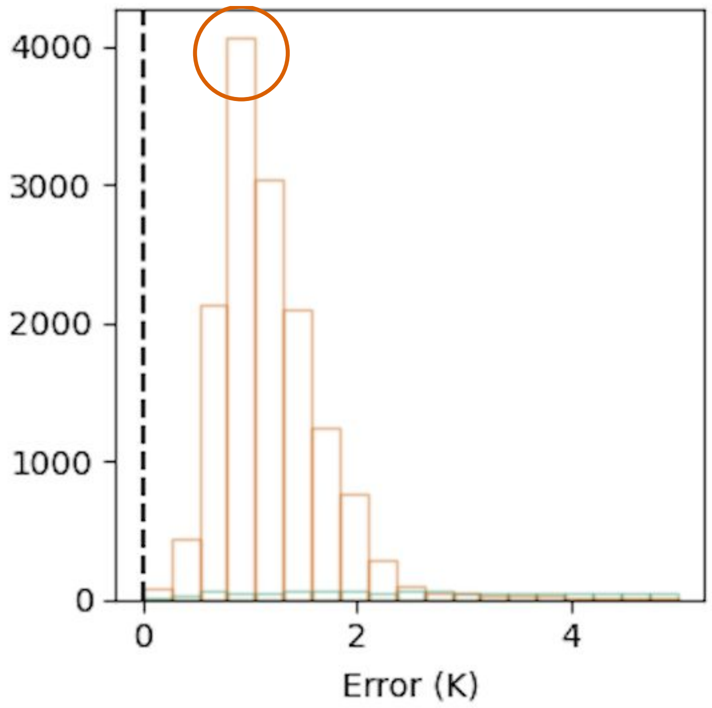
traditional λ_h biased 10% short

Quarter-period

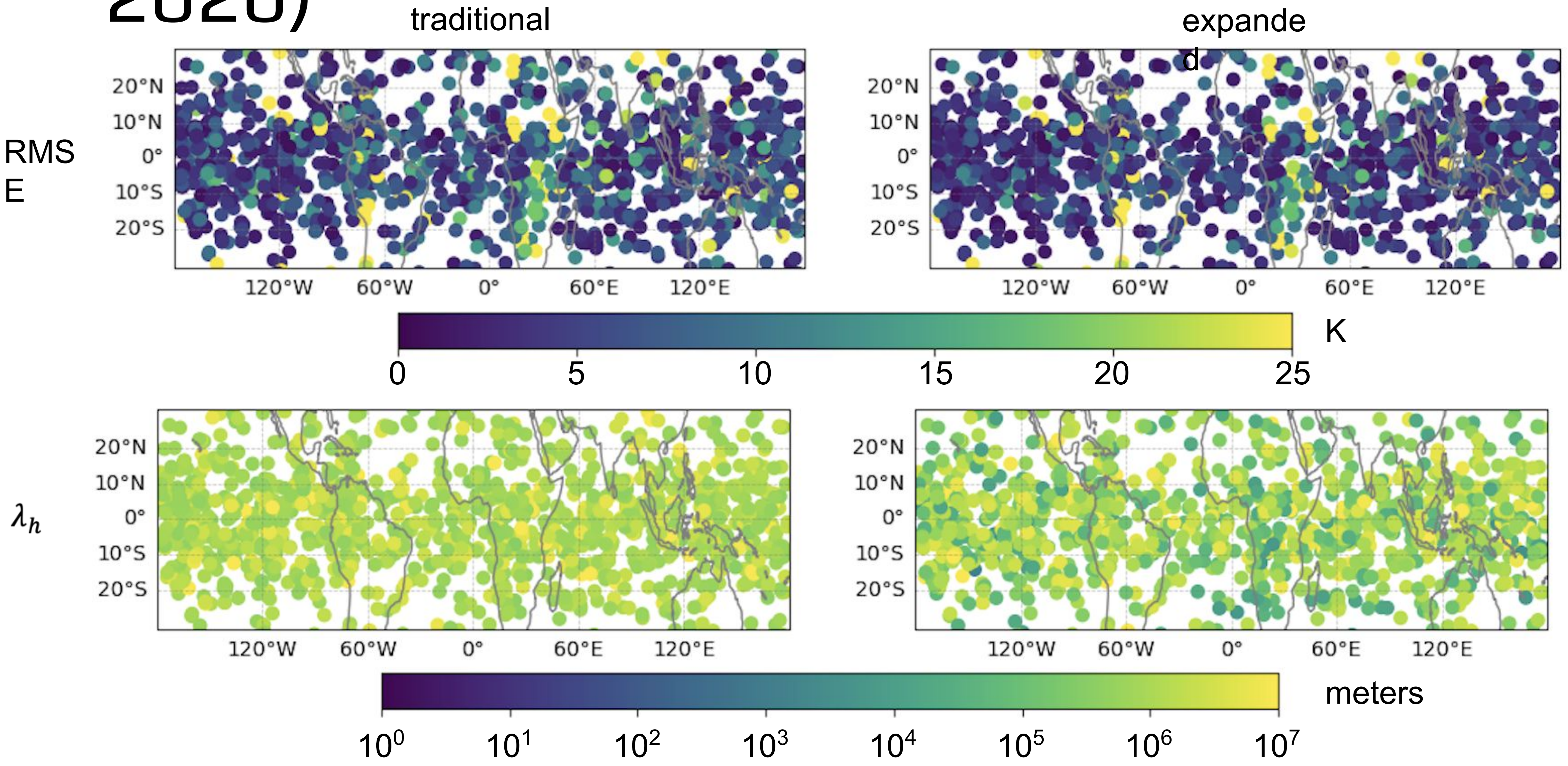


~200 triplets

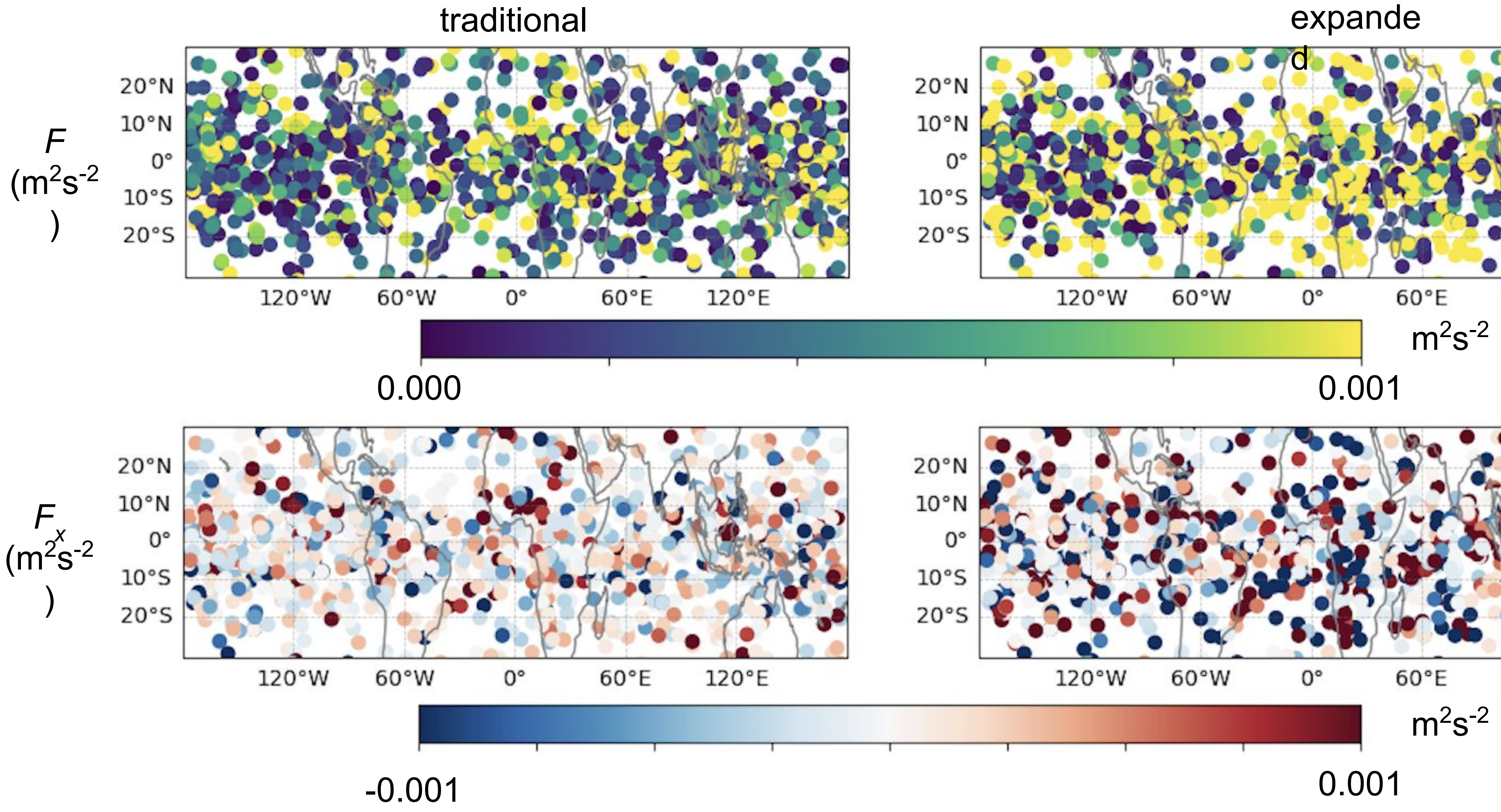
Quarter-period



Tighter fits, shorter wavelengths (Dec 2020)



Larger fluxes, questionable direction



Conclusion – We can better infer k, l

- Triads limited to ~~tens of minutes~~ hours apart
- Finite decomposition can step over "physical" wavenumber*
- Wave direction ~~ambiguous~~ can be guessed
- Susceptible to aliasing (but not as bad)
- ~~Difficult to~~ We evaluate goodness of fit

Needed improvements

- Remove background horizontal wind (better ω)
- Calculate N for each triplet (better ω)
- Generate gridpoint statistics over longer time periods
- Combine multiple GPS-RO datasets
- **Debug spectral transform algorithm**
- Perform calculations for each z, m